Solving Earth Gravity Field from GOCE Gravity Gradient Data by Tensor Spherical Harmonics Analysis

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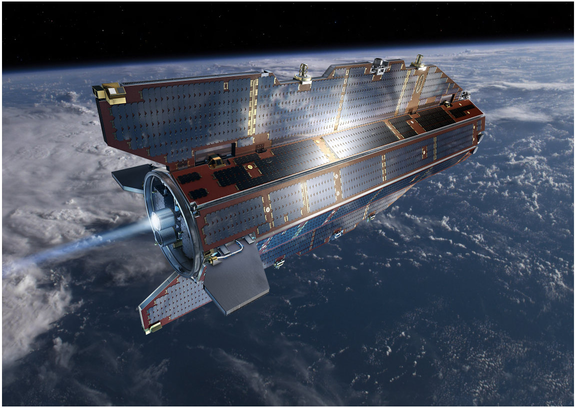
Outline

Simple Introduction of GOCE Satellite

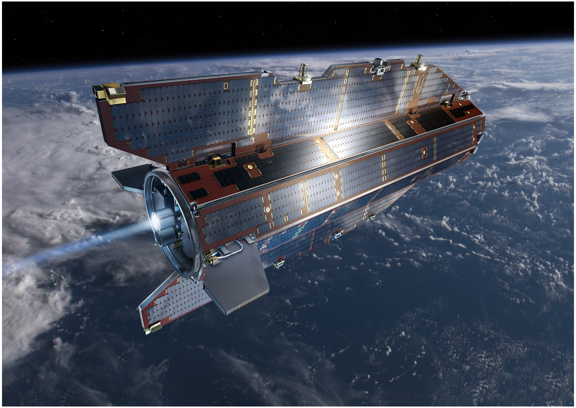
- Mode: SST + SGG



Picture cited from ESA

Simple Introduction of GOCE Satellite

- Mode: SST + SGG
- Average orbital height: 260 km



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Simple Introduction of GOCE Satellite

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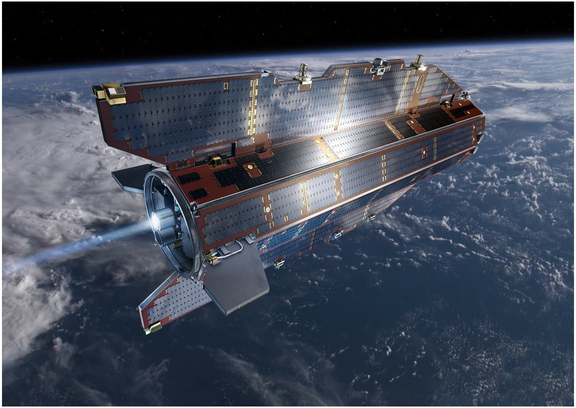
- Inclination: 96.7°



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Gravity Gradient Tensor (GGT)

Symmetric, trace-free

$$\boldsymbol{V} = \begin{pmatrix} V_{xx} & V_{xy} & V_{xz} \\ V_{yx} & V_{yy} & V_{yz} \\ V_{zx} & V_{zy} & V_{zz} \end{pmatrix}$$

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- Time-Wise Approach

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In most cases, V_{ij} $(i, j \in \{x, y, z\})$ **are treated independently.**

- Direct Approach
- Time-Wise Approach
- Space-Wise Approach
- $\cdots$

Expression of symmetric spherical dyadics

$$\begin{aligned}\boldsymbol{V} =& V_{zz}\mathbf{e}_{zz} + \\ & 2V_{xz}\mathbf{e}_{xz} + 2V_{yz}\mathbf{e}_{yz} + \\ & \frac{1}{2}(V_{xx} - V_{yy})(\mathbf{e}_{xx} - \mathbf{e}_{yy}) + 2V_{xy}\mathbf{e}_{xy} + \\ & \frac{1}{2}(V_{xx} + V_{yy})(\mathbf{e}_{xx} + \mathbf{e}_{yy})\end{aligned}$$

with

$$\mathbf{e}_{ij} = \mathbf{e}_i \otimes \mathbf{e}_j \qquad i, j \in \{x, y, z\}$$

Tensor Spherical Harmonics Analysis

Combinations of components (Rummel, 1992)

$$\boldsymbol{V}^{(1)} = V_{zz}\mathbf{e}_{zz}$$

$$\boldsymbol{V}^{(2)} = 2V_{xz}\mathbf{e}_{xz} + 2V_{yz}\mathbf{e}_{yz}$$

$$\boldsymbol{V}^{(3)} = \frac{1}{2}(V_{xx} - V_{yy})(\mathbf{e}_{xx} - \mathbf{e}_{yy}) + 2V_{xy}\mathbf{e}_{xy}$$

Series Representation of Combinations

Series representation (Martinec, 2003)

$$\boldsymbol{V}^{(1)} = \frac{GM}{R^3} \sum_{n=2}^{\infty} \left(\frac{R}{r}\right)^{n+3} \sum_{m=-n}^{n} (n+1)(n+2)\overline{C}_{nm}\mathbf{Z}_{nm}^{(1)}$$

$$\boldsymbol{V}^{(2)} = \frac{GM}{R^3} \sum_{n=2}^{\infty} \left(\frac{R}{r}\right)^{n+3} \sum_{m=-n}^{n} -2(n+2)\overline{C}_{nm}\mathbf{Z}_{nm}^{(2)}$$

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$\mathbf{Z}_{nm}^{(1)}, \mathbf{Z}_{nm}^{(2)}, \mathbf{Z}_{nm}^{(3)}$ are subsets of the complete orthogonal group of tensor functions.

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By a systematic application of the orthogonality, ...

Tensor Spherical Harmonics Analysis (TSHA)

Explicit expression (Martinec, 2003)

$$\overline{C}_{nm}^{(1)} = \frac{\mu}{(n+1)(n+2)} \iint V_{zz} Y_{nm} \mathrm{d}\sigma$$

$$\overline{C}_{nm}^{(2)} = \frac{\mu}{n(n+1)(n+2)} \iint \left[V_{xz} E_{nm} + V_{yz} F_{nm} \right] \mathrm{d}\sigma$$

$$\overline{C}_{nm}^{(3)} = \frac{\mu}{(n-1)n(n+1)(n+2)} \iint \left[(V_{xx} - V_{yy}) G_{nm} - 2 V_{xy} H_{nm} \right] \mathrm{d}\sigma$$

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- $\mu = \frac{R^3}{4\pi GM} \left(\frac{r}{R}\right)^{n+3}$

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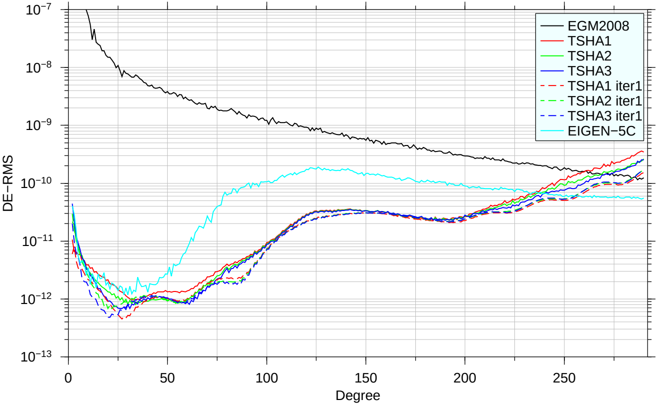
With

- $\mu = \frac{R^3}{4\pi GM} \left(\frac{r}{R} \right)^{n+3}$
- Y_{nm} are surface spherical harmonics.
- E_{nm}, F_{nm}, G_{nm} and H_{nm} are functions of Y_{nm}.

Numerical Experiment (No Noise)

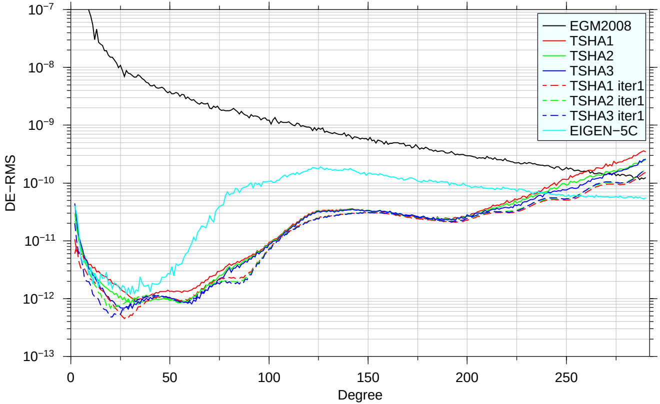
Simulation parameters



Numerical Experiment (No Noise)

Simulation parameters

- Gravity model: EGM2008



Numerical Experiment (No Noise)

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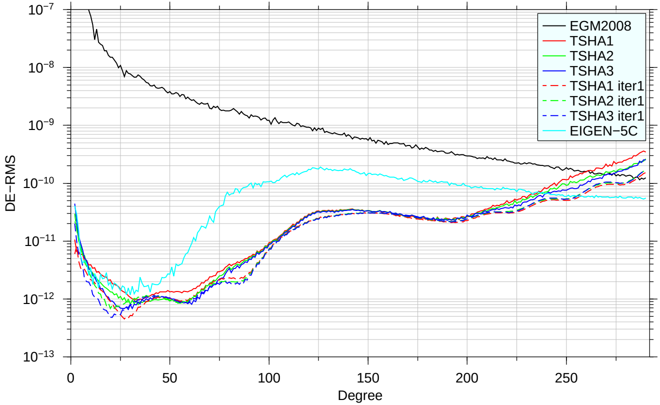

Numerical Experiment (No Noise)

Simulation parameters

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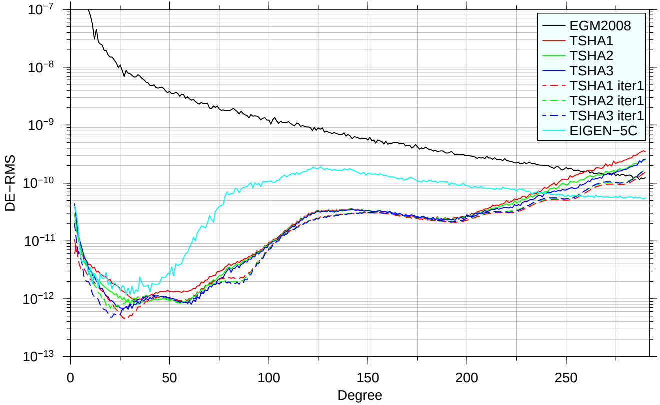
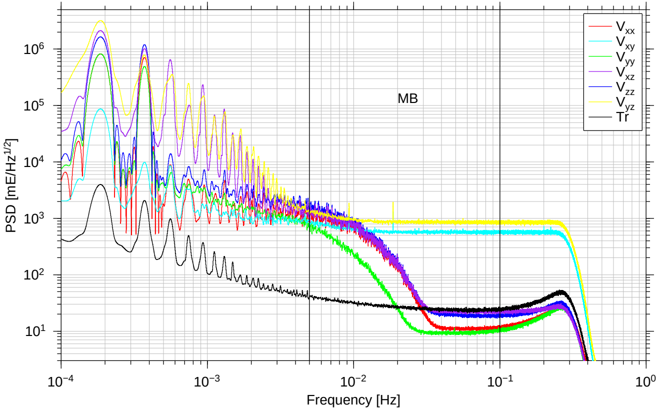
- Orbit: GOCE orbit from 20091101 to 20100111
- Model for continuation and filling: EIGEN-5C



The problems



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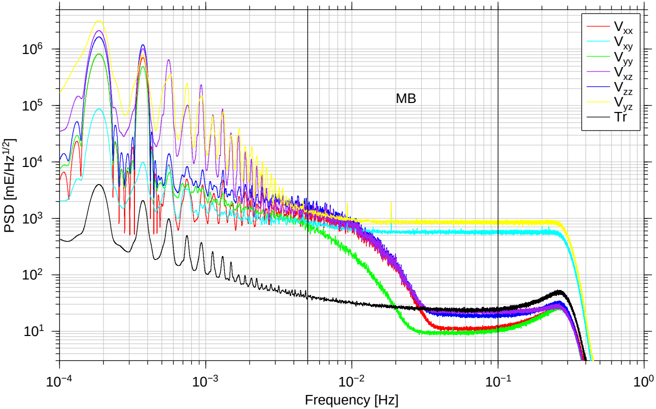

Problems

- Colored noise in GOCE gradiometric data

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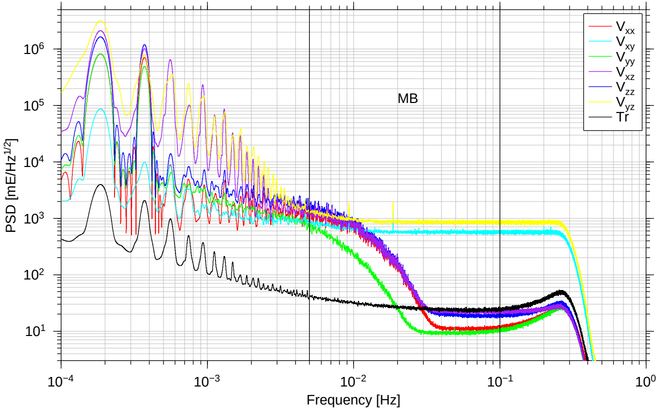

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- Ultra-low accuracy of V_{xy} and V_{yz}

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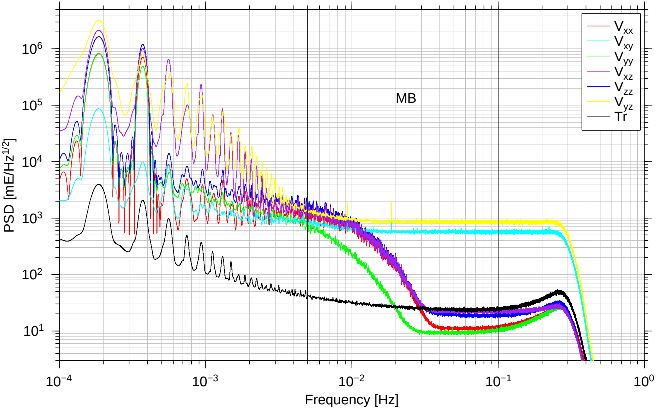

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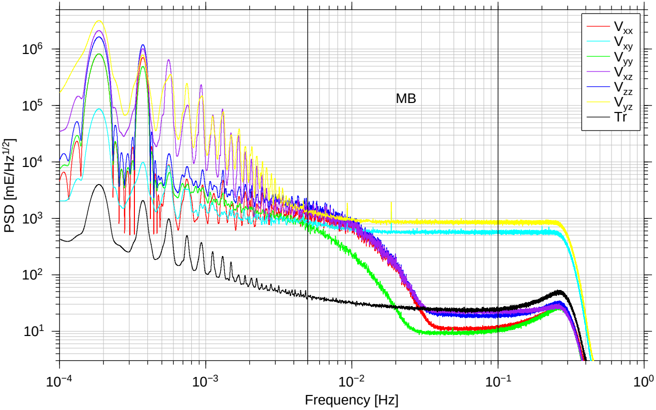

Problems

- Colored noise in GOCE gradiometric data
- Ultra-low accuracy of V_{xy} and V_{yz}
- The polar gap
- TSHA requires grid data on the sphere.

How to deal with the problems

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- Bandpass filter with zero-phase shift (Butterworth IIR, $5 \sim 100\,\mathrm{mHz}$)

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In addition

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- Reduce the griding error by remove-restore method

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- Bandpass filter with zero-phase shift (Butterworth IIR, $5 \sim 100\,\mathrm{mHz}$)
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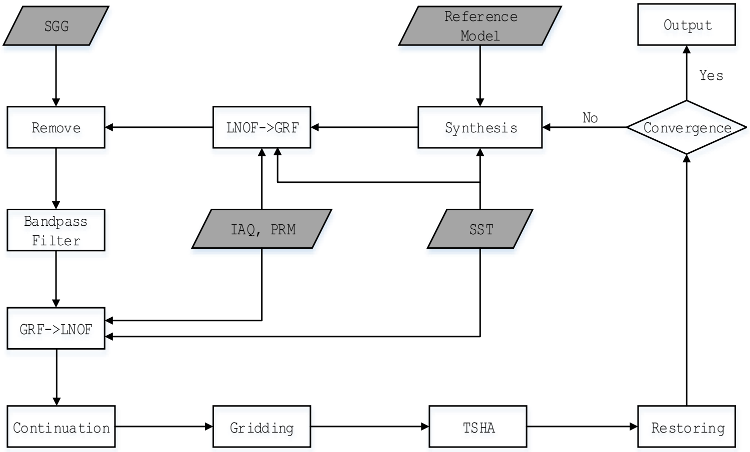
In addition

- Reduce the griding error by remove-restore method
- Reduce the influences of the prior model by iteration

Flow-chart


SGG
Reference Model
Output
Yes
Remove
LNOF->GRF
Synthesis
No
Convergence
Bandpass Filter
IAQ, PRM
SST
GRF->LNOF
Continuation
Gridding
TSHA
Restoring

Data and some parameters

Data

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- Period: November, 2009$\sim$June, 2010

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Parameters

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- Grid resolution: $10' \times 10'$

Data and some parameters

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- Period: November, 2009$\sim$June, 2010
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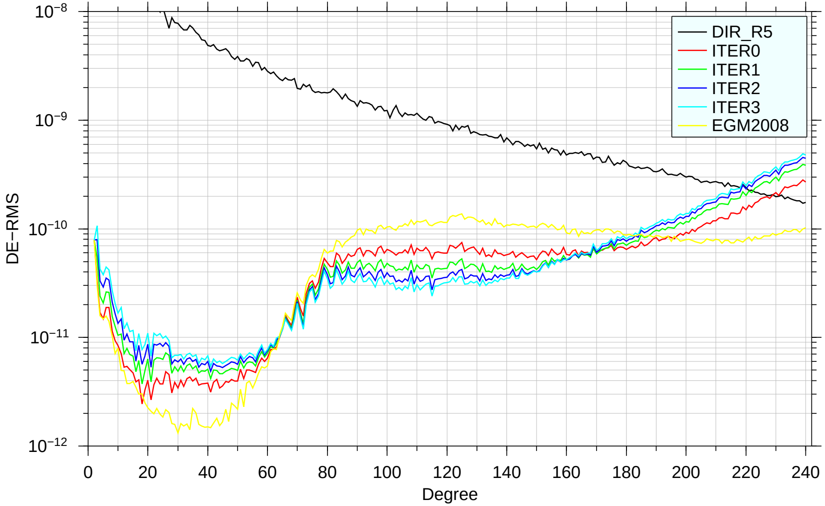
Parameters

- Reference model: EGM2008
- Grid resolution: $10' \times 10'$
- Degree: 240

Differences with Respect to DIR_R5 in Terms of Degree-Error Root Mean Square



Error Spectrum with Respect to DIR_R5 (log 10)

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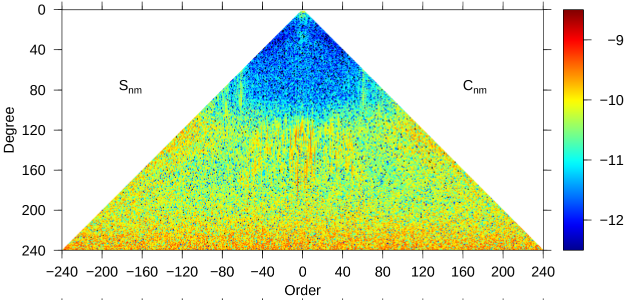

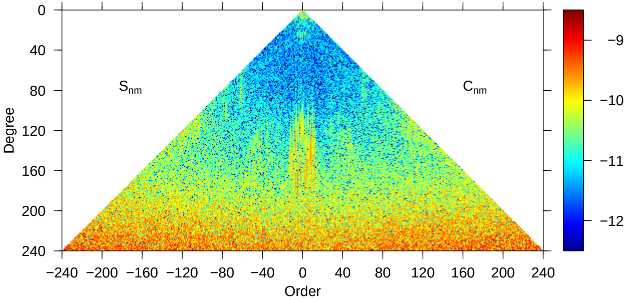

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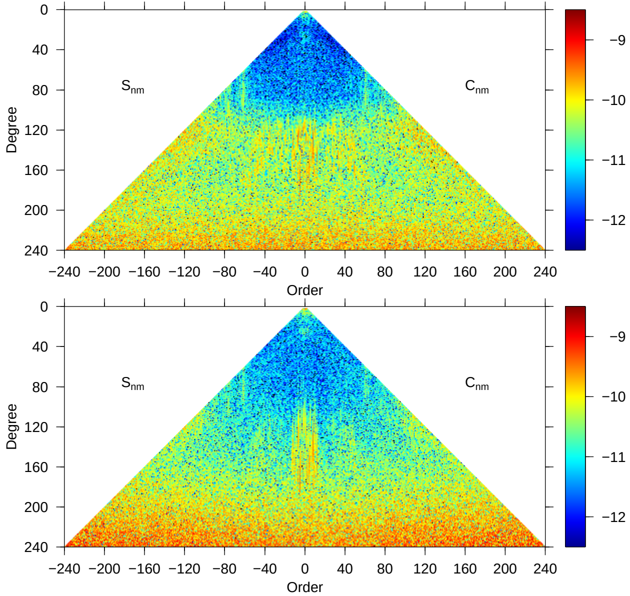
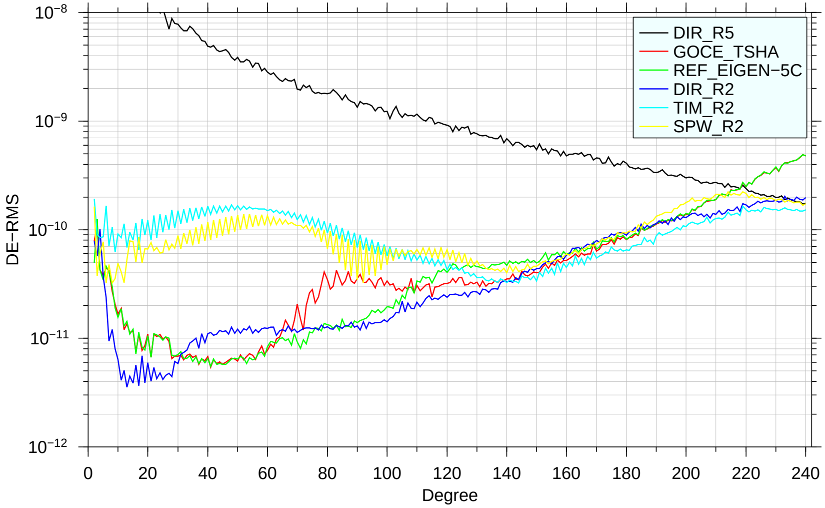


Initial solution

Final solution

Degree-Error RMS with Respect to DIR_R5



GPS/Levelling Validation in China (649 points) and America (6169 points)

Max degree 220 with the omission error compensated by EGM2008(Unit: m)

Region	Model	Max	Min	Mean	Std	RMS
China	TSHA	0.928	-0.956	0.234	0.206	0.311
	SPW_R2	1.038	-1.066	0.225	0.251	0.337
	DIR_R2	0.788	-0.637	0.235	0.189	0.301
	TIM_R2	0.809	-0.676	0.242	0.186	0.305
America	TSHA	0.395	-1.553	-0.514	0.318	0.604
	SPW_R2	0.493	-1.642	-0.501	0.319	0.594
	DIR_R2	0.269	-1.566	-0.509	0.297	0.590
	TIM_R2	0.320	-1.512	-0.510	0.300	0.592

- Slightly better than SPW_R2 in China

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Outlook

- All data during the entire mission preiod
- Wiener filter for filtering colored noise of gravity gradient data
- Block-diagonal LS for combination of 3 types of observations

Thank You!